

# Problem Set Four

Due date: Thursday March 13, 2007

1. **(20 pts)** Consider the following LP: minimize  $cx$  subject to  $Ax \leq b$ . Assume that the polytope described by  $AX \leq b$  is bounded. Prove that if there exists a feasible point  $x$  such that  $cx = a$ , then there exists a vertex  $x'$  such that  $cx' \leq a$ . Note that we proved a very similar but not identical statement in class.
  
2. **(20 pts)** Consider the following variant of the facility location problem: you are given a graph  $G = (V, E)$  with distances on the edges. There is a set of “clients”, each client is located in a certain node of the graph. (For simplicity, at most a single client per node.)  

The goal is to find where to place “servers” in the graph. Placing a server in node  $v$  costs  $c(v)$ . Once the servers are placed, each client is assigned to the closest server.

The optimization goal is to minimize the sum of costs we paid to place servers plus the sum, over all clients, of the distance between the client and the closest server.

Write IP describing this problem. Relax it to get LP. (Both should be polynomial size). State the dual of the resulting LP.
  
3. **(20 pts)** The goal of the following exercise is to devise an algorithm to assign tech support personnel to specific holidays. You are given a list of personnel ( $n$  persons) and a list of vacation periods (e.g. Thanksgiving, New Year, etc), each period spanning several contiguous vacation days. Let  $D_j$  be the set of days included in the  $j$ th vacation period. You need to produce a schedule satisfying the following constraints:
  - (a) For a given parameter  $c$ , each tech support person should be assigned to work at most  $c$  vacation days total.
  - (b) For each vacation period  $j$ , each person should be assigned to work at most one of the days during the period. In other words, although a person might be assigned to work on several vacation days, he or she should not be assigned to work more than one day during Thanksgiving weekend, more than one day on July 4th weekend, etc.
  - (c) Each vacation day should be assigned a single tech support person.
  - (d) For each person only certain vacation periods are viable. For example, a tech support person might request full vacation during Thanksgiving.

Describe a polynomial time algorithm to generate an assignment (or prove that one does not exist. Prove correctness. What is the running time of your algorithm ? [Hint: first try to solve without the 2nd constraint.]